Theorem

A is countable iff its elements can be listed, i.e. we can write

$$A = (a_1, a_2, \ldots, a_n),$$

or

$$A = (a_1, a_2, \ldots, a_n, \ldots).$$

Proof.

- If $|A| < \infty$, then easy. So assume $|A| = \infty$.
- If A is countable then there is $f : \mathbb{N} \to A$, which is bijective. And in particular surjective. Therefore the list

  $$(f(1), f(2), \ldots, f(n), \ldots)$$

  covers all of A.
- Conversely, if A can be listed, then this is implicitly a bijection to $\mathbb{N}$: given the list

  $$(a_1, a_2, \ldots, a_n, \ldots)$$

  of A, write $g(a_n) = n$, and this is a bijection.