Corollary 2

Theorem

$\mathbb{N}^p = \underbrace{\mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \cdots \times \mathbb{N}}_{p \text{ times}}$ *is countable.*

Proof.

- $p = 2$ we just did, and $p = 1$ by definition.
- Let $A_n = \{(a_1, a_2, \ldots, a_p, n), a_i \in \mathbb{N}\}$.
- The map $f\colon A_n \to \mathbb{N}^p$ that "forgets" the last entry is a bijection.
- And $\mathbb{N}^p = \bigcup_{n \in \mathbb{N}} A_n$.

□