Theorem

$\mathbb{N}^2 = \mathbb{N} \times \mathbb{N}$ *is countable.*

Proof.

Let $A_n = \{(k, n), k \in \mathbb{N}\}$, i.e.

$$A_n = \{(1, n), (2, n), (3, n), \dots\}.$$

The map $f \colon A_n \to \mathbb{N}$ where

$$f((i, n)) = i \tag{2}$$

is a bijection, so A_n is countable.
But $\mathbb{N} \times \mathbb{N} = \bigcup_{n \in \mathbb{N}} A_n$.

$\square$