Summation formula

Theorem

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

Another proof.

- Let $|A| = n$, and let $B = A \cup \{\star\}$.
- Let $C \subseteq B$, $|C| = k$. Then either $\star \in C$ or $\star \notin C$.
- If $\star \notin C$, then C is a subset of A of size k;
- If $\star \in C$, then C is a subset of A of size $(k-1)$ plus $\star$;
- There are $\binom{n}{k}$ options in case 1 and $\binom{n}{k-1}$ options in case 2.

$\square$