Theorem

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

One proof.

$$\binom{n}{k} + \binom{n}{k-1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-(k-1))!}$$

$$= \frac{(n-k+1)n!}{k!(n-k+1)!} + \frac{k \cdot n!}{k!(n-k+1)!} = \binom{n+1}{k}$$

$\square$