Lemma

If $0 < k < p$, then $\binom{p}{k}$ is a multiple of p.

Proof.

- $\binom{p}{k}$ is an integer, call it m.
- Recall that
$$\binom{p}{k} = \frac{p!}{k!(p-k)!} = m.$$
- This gives
$$p! = m \cdot k! \cdot (p-k)!$$
- The LHS is divisible by p.
- Since $k < p$, $k!$ is not divisible by p;
- Since $k - p < p$, $k!$ is not divisible by p;
- Therefore m is divisible by p.