Well-defined 2

Definition

Let X be set, $\sim$ an equivalence relation on X, and $X/\sim$ the partition generated by $\sim$.

- Let $f \colon X \to X$ be any function.
- Let us **try** to define $\widetilde{f} \colon X/\sim \to X/\sim$ by

$$\widetilde{f}([x]) = [f(x)].$$

- The only way this is well-defined is if it gives the same value for every element in $[x]$.

Definition

We say that $\widetilde{f}$ is **well-defined** if

$$x \sim y \implies f(x) \sim f(y).$$