- Let $X = \mathbb{N}$, choose

$$S = \{1, 4, 9, 16, \dots\}, \quad T = \mathbb{N} \setminus S.$$

- $S =$ "perfect squares" and $T =$ "not squares".
- Clearly a partition:

$$S \cup T = \mathbb{N}, \quad S \cap T = \emptyset.$$

- So what is $S + S$?
- Take $9, 16 \in S$. Then $9 + 16 = 25 \in S$.
- But we also have $16, 36 \in S$, and $16 + 36 = 52 \notin S$.

In particular, $S + S$ is neither S nor T, so this operation is not consistent with the partition!