

- Let $f: \mathbb{Z}_4 \rightarrow \mathbb{Z}_6$, with $f(x) = x$.

- Note that

$$2 \equiv 6 \pmod{4}, \text{ but } 2 \not\equiv 6 \pmod{6}.$$

- So we have $2 \sim 6$ in \mathbb{Z}_4 , but $f(2) \not\sim f(6)$ in \mathbb{Z}_6 .