Example

- Consider the set $S = \mathbb{Z} \times \mathbb{N}$, and define a relation on S:

$$(a, b) \sim (c, d) \iff ad = bc.$$

- Check that this is an equivalence relation:
  - **Reflexive.** Since $ab = ab$, we have $(a, b) \sim (a, b)$.
  - **Symmetric.**
    $(a, b) \sim (c, d) \iff ad = bc \iff cb = ad \iff (c, d) \sim (a, b)$.
  - **Transitive.** Assume

$$(a, b) \sim (c, d) \wedge (c, d) \sim (e, f)$$

This means

$$ad = bc \wedge cf = ed.$$

Then

$$\begin{aligned} bcf &= adf \\ bed &= adf \\ be &= af, \end{aligned}$$

so $(a, b) \sim (e, f)$.