Theorem

The operations $+_n, *_n$ are well-defined.

Proof.

We need to show that if x, x' are in the same equivalence class, and y, y' are in the same equivalence class, then $x + x'$ and $y + y'$ are in the same equivalence class.
But note:

$$x \sim x' \iff x = x' + kn, \quad y \sim y' \iff y = y' + \ell n,$$

so

$$(x + y) - (x' + y') = (k + \ell)n$$

so $x + y, x' + y'$ are in the same equivalence class.

□