Proof of part 2.

Let $(A_n)n \in I$ be a partition of X, and we say $x\mathcal{R}y$ if and only if there is a single A_n such that x, y both in A_n.
We will show that $\mathcal{R}$ is an equivalence relation.

- **refl.** Let $x \in X$. Then $x \in A_n$ for some n, and thus $x\mathcal{R}x$.
- **sym.** Let $x\mathcal{R}y$. Then by definition there is an n such that $x, y \in A_n$, but then $y, x \in A_n$ and $y\mathcal{R}x$.
- **trans.** Let $x\mathcal{R}y$ and $y\mathcal{R}z$.
    - Since $x\mathcal{R}y$, there is A_k with $x, y \in A_k$.
    - Since $y\mathcal{R}z$, there is A_ℓ with $y, z \in A_\ell$.
    - Since $y \in A_k$ and $y \in A_\ell$, $y \in A_k \cap A_\ell$ and thus $A_k \cap A_\ell \neq \emptyset$.
    - Thus $A_k = A_\ell$ (partition!!) and $x, y, z \in A_k$ so $x\mathcal{R}z$.

$\square$