

Proof of part 1.

Let \sim be an equivalence relation on X , and consider the collection

$$\{[x] : x \in X\}.$$

We need to show two things:

- 1 $\bigcup_{x \in X} [x] = X$ (but this follows since $x \in [x]$);
- 2 For any $x, y \in X$, either $[x] = [y]$, or $[x] \cap [y] = \emptyset$.
 - Assume $[x] \cap [y] \neq \emptyset$. Take $z \in [x] \cap [y]$.
 - Then $z \sim x \wedge z \sim y$
 - By symmetry and transitivity, $x \sim y$
 - By lemma, $[x] = [y]$.

