Theorem

Let X be any set.

1. *If $\sim$ is an equivalence relation on X, the equivalence classes of $\sim$ form a partition of X.*
2. *Let $\{A_n : n \in I\}$ be a partition of X. Define a relation $\mathcal{R}$ on X where*

$$x\mathcal{R}y \iff \exists n \in I, x \in A_n \wedge y \in A_n.$$

   Then $\mathcal{R}$ is an equivalence relation.

Basically we have this one-to-one correspondence:

$$\{\text{partitions}\} \leftrightarrow \{\text{equivalence relations}\}.$$