Lemma

Say that $\sim$ *is an equivalence relation. Then*

$$x \sim y \iff [x] = [y].$$

Proof.

$\implies$

- Assume $x \sim y$. Let $z \in [x]$, then $z \sim x$.
- By transitivity, $z \sim y$ and thus $z \in [y]$. Therefore $[x] \subseteq [y]$.
- Now let $z \in [y]$, so that $z \sim y$.
- By transitivity (and symmetry), $z \sim x$ and thus $z \in [x]$. Therefore $[y] \subseteq [x]$.

$\impliedby$

Assume $[x] = [y]$.
$x \in [x]$ by definition, and since $[x] = [y]$, we have $x \in [y] \implies x \sim y$. $\square$