Another example of ER and partition

- Consider $X = \mathbb{R}$ and the partition $\{A_n\}$ with $A_n = [n, n+1)$.
- Consider the function $f : \mathbb{R} \to \mathbb{Z}$, where

$$f(x) = n \iff x \in A_n \iff n \leq x < n+1.$$

  (f is also called the "floor" function)
- Notice that

$$\{x \in \mathbb{R} : f(x) = n\} = A_n$$

- So let us define a relation

$$x\mathcal{R}y \iff f(x) = f(y).$$

- We showed on last in-class activity that $\mathcal{R}$ is an equivalence relation...

In general, given a partition $(A_n)_{n\in I}$ of X, defining $f : X \to I$

$$f(x) = n, \quad \forall x \in A_n$$

and

$$x\mathcal{R}y \iff f(x) = f(y).$$

always gives an equivalence relation.