Example of ER and partition

Let $A = \mathbb{Z}$ and

$$x\mathcal{R}y \iff (y - x) \text{ is multiple of } n \iff x \equiv y \pmod{n}$$

- **reflexive**: $x - x = 0$ is multiple of n for all $x \in \mathbb{Z}$;
- **symmetric**: if $y - x = kn$, then so is $x - y = -(y - x) = -kn$;
- **transitive**: assume that $x - y = kn$ and $y - z = \ell n$ are multiples of n. Then

  $$x - z = (x - y) + (y - z) = (k + \ell)n$$

  is also multiple of n.

So what are the equivalence classes of this equivalence relation?

$$[r] = \{y \in \mathbb{Z} : y \sim r\} = \{y \in \mathbb{Z} : y \equiv r \pmod{n}\} = A_r.$$