- $A = \mathbb{R}$, we say $x\mathcal{R}y \iff x^2 = y^2$
  - reflexive: $x^2 = x^2$
  - symmetric: if $x^2 = y^2$, then $y^2 = x^2$;
  - transitive: if $x^2 = y^2$ and $y^2 = z^2$, then $x^2 = z^2$.
- $A = \mathbb{R}$, where $x\mathcal{R}y \iff |y - x| \leq 1$.
  - reflexive: $|x - x| = 0 < 1$;
  - symmetric: Since $|x - y| = |y - x|$, if $|y - x| \leq 1$ then $|x - y| \leq 1$;
  - transitive: Now assume that $x \sim y$ and $y \sim z$. Then we have

    $$|x - z| = |(x - y) + (y - z)| \leq |x - y| + |y - z| \leq 1 + 1 = 2,$$

    and this suggests that it is not transitive. In fact, if we choose

    $$x = 0, \quad y = 1, \quad z = 2,$$

    then

    $$x \sim y, \quad y \sim z, \quad x \not\sim z. \tag{1}$$

    Not an equivalence relation!!