Examples 1

• The diagonal relation on A:

$$\mathcal{R} = \{(a, a) : a \in A\}.$$

• Let $A = \mathbb{Z}$ and

$$x\mathcal{R}y \iff (y-x) \text{ is even } \iff x \equiv y \pmod{2}$$

- reflexive: x x = 0 is even for all $x \in \mathbb{Z}$;
- symmetric: if y x is even, then so is x y = -(y x);
- transitive: assume that x y and y z are even. Then

$$x - z = (x - y) + (y - z)$$

is also even.