Checking condition 2

2. Assume that if $(y, z_1) \in f^*$ and $(y, z_2) \in f^*$, then $z_1 = z_2$.

- As argued before, this means that $y = f(x)$ for some x.
- Say $x_1, x_2 \in A$ with $f(x_1) = f(x_2) = y$.
- Then $(y, x_1) \in f^*$ and $(y, x_2) \in f^*$.
- For f^* to be a function, this requires that $x_1 = x_2$, but we have shown that $f(x_1) = f(x_2) \implies x_1 = x_2$, which is the definition of injective.

Therefore

$$f^* \text{ is a function} \implies f \text{ is injective.}$$