

Checking condition 1

④ Assume that $\forall y \in B$, the pair (y, \cdot) occurs in f^* at least once;

• Since

$$f^* = \{(f(x), x) : x \in A\},$$

• this means that for all $y \in B$, there is some $x \in A$ with $f(x) = y$.

• But this is the same as saying that f is surjective.

Therefore

f^* is a function $\implies f$ is surjective.