

Proof.

- Recall that if we think of f as a relation $f \subseteq A \times B$, then

$$f = \{(x, f(x)) : x \in A\},$$

and

$$f^* = \{(f(x), x) : x \in A\}.$$

- What do we need for this to be a function from B to A ? We need:
- For any $y \in B$, there is exactly one pair $(y, \cdot) \in f^*$, i.e.
 - $\forall y \in B$, the pair (y, \cdot) occurs in f^* at least once;
 - if $(y, z_1) \in f^*$ and $(y, z_2) \in f^*$, then $z_1 = z_2$.

