- So we have shown that

$$\begin{aligned} f^* \text{ is a function } &\implies (f \text{ is injective}) \wedge (f \text{ is surjective}) \\ &\implies f \text{ is bijective.} \end{aligned}$$

- This is one direction of the proof, but we showed directly in the last lecture that if f is bijective, then f^* is a function.
- Therefore f^* is a function iff f is bijective.

Recalling that

$$f^* = \{(f(x), x) : x \in A\},$$

and noting two things:

- $f^{-1}(f(x)) = x$ for all $x \in A$
- the range of f is all of B, and therefore

$$f^* = \{y, f^{-1}(y) : y \in B\}$$

is the graph of f^{-1}, whenever f is invertible.