Theorem

Let $\mathcal{R}, \mathcal{S} \subseteq A \times B$. Then

1. $(\mathcal{R}^*)^* = \mathcal{R}$;
2. $\mathcal{R} \subseteq \mathcal{S} \implies \mathcal{R}^* \subseteq \mathcal{S}^*$
3. $(\mathcal{R} \cup \mathcal{S})^* = \mathcal{R}^* \cup \mathcal{S}^*$.
4. $(\mathcal{R} \cap \mathcal{S})^* = \mathcal{R}^* \cap \mathcal{S}^*$.
5. *If $A = B$, then*
    - $\mathcal{R} \cup \mathcal{R}^*$
    - $\mathcal{R} \cap \mathcal{R}^*$

   are both symmetric relations.

Proof of 5.

Using [3] and [1], we have

$$(\mathcal{R} \cup \mathcal{R}^*)^* = \mathcal{R}^* \cup (\mathcal{R}^*)^* = \mathcal{R}^* \cup \mathcal{R} = \mathcal{R} \cup \mathcal{R}^*,$$

and similarly for the intersection.

□