

- 1 Recall

$$\mathcal{R} = \{(x, y) \in \mathbb{R}^2 : x < y\}.$$

Then

$$\mathcal{R}^* = \{(y, x) \in \mathbb{R}^2 : x < y\} = \{(a, b) \in \mathbb{R}^2 : a > b\}$$

So the conjugate of “<” is “>” (and vice versa)

- 2 Let's say that $f: A \rightarrow B$ is an **invertible** function, and define

$$\mathcal{R} = \{(x, f(x)) : x \in A\}. \quad (1)$$

Then

$$\mathcal{R}^* = \{(f(x), x) : x \in A\}.$$

But note two things:

- $f^{-1}(f(x)) = x$ for all $x \in A$
- the range of f is all of B , and therefore

$$\mathcal{R}^* = \{y, f^{-1}(y) : y \in B\}$$

is the graph of f^{-1} .