

- Let

$$A_r = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = r^2\}$$

This is the circle of radius r .

- Claim: every $x \in \mathbb{R}^2$ is in exactly one A_r for $r \geq 0$.
- Let $r, s \geq 0$ with $r \neq s$. Then $r^2 \neq s^2$. Let $x \in A_r$, and then

$$x^2 + y^2 = r^2 \neq s^2 \implies x \notin A_s,$$

and vice versa.

- For any $(x, y) \in \mathbb{R}^2$, we have

$$(x, y) \in A_{\sqrt{x^2+y^2}}.$$