

## Theorem

For all  $n \in \mathbb{N}$ , let

$$A_n = [n, n + 1) = \{x \in \mathbb{R} : n \leq x < n + 1\}.$$

Then we have two claims:

- The  $A_n$  are pairwise disjoint;
- $\bigcup_{i \in \mathbb{N}} A_i = \mathbb{R}$ .

## Proof for disjoint.

- If  $m \neq n$ , say  $m < n$ .
- $x \in A_m \implies x < m + 1 \leq n \implies x < n \implies x \notin A_n$ .

