- We have

$$x \in \bigcap_{i \in \mathbb{N}} A_i \iff 0 \leq x < 1/i \quad \forall i \in \mathbb{N}.$$

  - We see that the number 0 satisfies each of those inequalities.
  - We claim that no positive number satisfies all of them.
  - Pick $x > 0$. Choose $n > 1/x$, then $x > 1/n$,
  - and then $x \notin A_n \implies x \notin \cap_{i \in \mathbb{N}} A_i$.

- Therefore:

$$\bigcap_{i \in \mathbb{N}} A_i = \{0\}.$$

- Note! Here is something that looks ok but is not:

$$\bigcap_{i=1}^{\infty} A_i = \lim_{M \to \infty} \bigcap_{i=1}^{M} A_i = \lim_{M \to \infty} A_M = \lim_{M \to \infty} [0, 1/M) = [0, 0)$$

This kind of ends up being nonsense... so be careful!