A trickier example

- Let $A_n = [0, 1/n) = \{x \in \mathbb{R} : 0 \leq x < 1/n\}$ for all $n \in \mathbb{N}$.
- Note that if $m > n$, then $A_m \subsetneq A_n$. (The collection is **decreasing**.)
- We see that
$$\bigcup_{i \in \mathbb{N}} A_i = A_1 = [0, 1).$$
  - Why?
  - Since $A_k \subseteq A_1$ for all $k \in \mathbb{N}$,
  - if $x \in \cup_{i \in \mathbb{N}} A_i$ then $x \in A_k$ for some k and thus $x \in A_1$.
- Now we consider
$$\bigcap_{i \in \mathbb{N}} A_i.$$
- Then
$$x \in \bigcap_{i \in \mathbb{N}} A_i \iff 0 \leq x < 1/i \quad \forall i \in \mathbb{N}.$$