- We want to show
$$\bigcup_{f \in C_0(\mathbb{R})} A_f = \mathbb{R}$$
    - Let $z \in \mathbb{R}$, we want to show that $z \in A_f$ for some $f \in C_0(\mathbb{R})$, or
    - for any $z \in \mathbb{R}$, there is $f \in C_0(\mathbb{R})$ with $f(z) = 0$.
    - Pick $f(x) = x(x - z)$.
- We want to show
$$\bigcap_{f \in C_0(\mathbb{R})} A_f = \{0\}.$$
    - Since $f(0) = 0$ for all $f \in C_0(\mathbb{R})$, we have $\{0\} \in A_f$ for all f.
    - Now let $z \neq 0$.
    - Note that $g(x) = x$ is in $C_0(\mathbb{R})$, but if $z \neq 0$, then $g(z) \neq 0$.
    - Therefore $z \notin A_g$ and $z \notin \bigcap_{f \in C_0(\mathbb{R})} A_f = \{0\}$.