Example with a pretty crazy index set

- Let $C_0(\mathbb{R}) = \{ f : \mathbb{R} \to \mathbb{R} | f \text{ is continuous } \wedge f(0) = 0 \}.$
- For any $f \in C_0(\mathbb{R})$, we define the set

$$A_f = \{z \in \mathbb{R} : f(z) = 0\}.$$

Then

$$\bigcup_{f\in C_{\boldsymbol{0}}(\mathbb{R})}A_f=\mathbb{R},\quad \bigcap_{f\in C_{\boldsymbol{0}}(\mathbb{R})}A_f=\{0\}.$$