Proof 3

Induction.

- $n = 1$: If $A = \{x\}$, then $\mathcal{P}(A)$ has two elements: $\emptyset$ and $\{x\}$.
- Now assume theorem true for all sets of size k, and let D be a set of size $k+1$:
$$D = \{d_1, d_2, \dots, d_{k+1}\}.$$
- Let $A = D \setminus \{d_{k+1}\} = \{d_1, d_2, \dots, d_k\}$, and so $|A| = k$.
- By induction hypothesis, $|\mathcal{P}(A)| = 2^k$.
- Note that any subset of A gives rise to **two** subsets of D:
$$A, \quad A \cup \{d_{k+1}\}.$$
- Therefore
$$|\mathcal{P}(D)| = 2\,|\mathcal{P}(A)| = 2 \cdot 2^k = 2^{k+1}.$$

□