Theorem

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

Proof.

Let $|A| = n$. Then

$$2^n = \#\{\text{subsets of } A\} = \sum_{k=0}^{n} \#\{\text{subsets of } A \text{ of size } k\} = \sum_{k=0}^{n} \binom{n}{k}.$$

□