Theorem

For all $n \in \mathbb{N}$*, n can be written as a sum of* distinct *powers of* 2*, i.e.*

$$n = 2^{r_1} + 2^{r_2} + \cdots + 2^{r_q},$$

with the r_i *all different.*

Proof.

- $n = 1$: $1 = 2^0$. Check.
- Fix k and assume true for all numbers up to and including k.
- Consider $k+1$. Let ℓ be maximal so that $2^\ell \leq k+1$.
- Define $m = k + 1 - 2^\ell$. Since $2^\ell > 0$, $m \leq n$.
- By assumption,
$$m = 2^{r_1} + 2^{r_2} + \cdots + 2^{r_q}.$$
- So
$$n = 2^\ell + 2^{r_1} + 2^{r_2} + \cdots + 2^{r_q}$$
- But what if $\ell = r_j$ for some j? Then $m \geq 2^\ell$, and $k + 1 = m + 2^\ell \geq 2^{\ell+1}$. This contradicts maximality of ℓ.

□