

Definition

We say $n \in \mathbb{N}$, $n \geq 2$ is **prime** if the only factors are 1 and n .

If n is not prime, then we say it is **composite**.

Specifically, if n is composite then $n = a \cdot b$ with $1 < a, b < n$.

Theorem

Every $n \in \mathbb{N}$ with $n \geq 2$ is a product of primes.

(Here we think of a prime as a “product of length 1”.)

Proof.

- Base case: $n = 2$ is prime.
- Now fix k , and assume true for all numbers $\leq k$.
- Choose $k + 1$.
- If $k + 1$ is prime, done.
- If $k + 1$ is composite, then $k + 1 = a \cdot b$ with $a, b < k + 1$, and therefore $a, b \leq k$, so both are a product of primes. Concatenate!

