Theorem

$$\forall n \in \mathbb{N}, \quad G_n < (a+b)^n.$$

Proof.

- We use strong induction. First note that if $a, b \geq 1$ then $(a+b) \geq 2$.
- Let us first use base cases: $n = 1$ and $n = 2$.
- $n = 1$: $G_1 = 1 < 2 \leq (a+b)^1$.
- $n = 2$: $F_2 = 1 < 4 \leq (a+b)^2$.
- Now fix k and assume that theorem holds for all $j \leq k$.
- Then

$$\begin{aligned} G_{k+1} &= G_k + G_{k-1} \\ &< (a+b)^k + (a+b)^{k-1} \\ &< (a+b)^k + (a+b)^k \\ &= 2 \cdot (a+b)^k \leq (a+b)^{k+1}. \end{aligned}$$

□