Gibonacci numbers

Pick $a, b \geq 1$. Let us define a sequence by:

- $G_1 = 1$,
- $G_2 = 1$,
- $\forall n > 2, G_n = aG_{n-1} + bG_{n-2}$.

For $a = 2, b = 3$, some terms in this sequence are

$$1, 1, 5, 17, 61, 317, \ldots$$

Theorem

$$\forall n \in \mathbb{N}, \quad G_n < (a + b)^n.$$