Theorem

$$\forall n \in \mathbb{N}, \quad F_n < 2^n.$$

Proof.

- We use strong induction.
- Let us first use base cases: $n = 1$ and $n = 2$.
- $n = 1$: $F_1 = 1 < 2^1$.
- $n = 2$: $F_2 = 1 < 2^2$.
- Now fix k and assume that theorem holds for all $j \leq k$.
- Then

$$\begin{aligned} F_{k+1} &= F_k + F_{k-1} \\ &< 2^k + 2^{k-1} \\ &< 2^k + 2^k \\ &= 2 \cdot 2^k = 2^{k+1}. \end{aligned}$$

□