

- We can also start at an integer greater than one ( a different “base case”) as long as we move to the right of our base case.
- For example, let us try and prove that

$$\forall n \geq 4, 3^n > n^3.$$

- Check  $n = 4$ :  $3^4 = 81, 4^3 = 64$ .
- Assume that  $3^k > k^3$ .
- Note that

$$(k+1)^3 = \left(\frac{k+1}{k}\right)^3 k^3 = \left(1 + \frac{1}{k}\right)^3 k^3 \geq \left(\frac{5}{4}\right)^3 k^3 \approx 1.95 * k^3 < 2k^3.$$

- Then

$$3^{k+1} = 3 \cdot 3^k > 2 \cdot 3^k > 2 \cdot k^3 > (k+1)^3.$$