- To prove: $\forall n \in \mathbb{N}, S_n = (-1)^n \frac{n(n+1)}{2}$.
- Base case: $S_1 = 1$ and $\frac{1 \cdot 2}{2} = 1$.
- Now assume that

$$S_k = (-1)^k \frac{k(k+1)}{2},$$

and note that

$$S_{k+1} = S_k + (-1)^{k+1}(k+1)^2.$$

- Then we compute

$$\begin{aligned} S_{k+1} &= S_k + (-1)^{k+1}(k+1)^2 \\ &= (-1)^k \frac{k(k+1)}{2} + (-1)^{k+1}(k+1)^2 \\ &= (-1)^k \frac{(k+1)}{2}[k - 2(k+1)] = (-1)^k \frac{(k+1)}{2}(-1)(k+2) \\ &= (-1)^{k+1} \frac{(k+1)(k+2)}{2}. \end{aligned}$$