- One more example:

$$S_n = 1 + 2 + 4 + 8 + 16 + \cdots + 2^n = \sum_{i=0}^{n} 2^i.$$

- We compute a few:

$$S_1 = 1, \quad S_2 = 3, \quad S_3 = 7, \quad S_4 = 15, \ldots$$

  so a reasonable guess is

$$S_n = 2^{n+1} - 1.$$

- Note

$$S_{n+1} = S_n + 2^{n+1}$$

- Check: $S_1 = 1 + 2 = 3$, and $2^2 - 1 = 4 - 1 = 3$.
- Now assume $S_k = 2^{k+1} - 1$, then:

$$\begin{aligned} S_{k+1} &= S_k + 2^{k+1} = 2^{k+1} - 1 + 2^{k+1} \\ &= 2 \cdot 2^{k+1} - 1 = 2^{k+2} - 1. \end{aligned}$$