

- We have the formula that

$$T_n = \sum_{i=1}^n i = 1 + 2 + 3 + \dots + (n-1) + n.$$

- We want to show that

$$\forall n \in \mathbb{N}, T_n = \frac{n(n+1)}{2}.$$

- We check for $n = 1$:

$$T_1 = 1, \quad \frac{1 \cdot 2}{2} = 1.$$

- Now notice that

$$T_{n+1} = T_n + (n+1).$$

- Now assume that

$$T_k = \frac{k(k+1)}{2},$$

and then we compute

$$T_{k+1} = \frac{k(k+1)}{2} + (k+1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}.$$

- This is the formula with $k+1$ plugged in for n