## Corollary

*Let $A, B$ be finite. Then*

$$|A| = |B| \iff \text{there exists a bijection } f\colon A \to B.$$

## Proof.

$\boxed{\implies}$

Assume that $|A| = |B|$.
Since $|A| \leq |B|$, there is an injective function $f\colon A \to B$.
Let $A = \{a_1, a_2, \dots, a_n\}$
Consider the set

$$\{f(a_1), f(a_2), \dots, f(a_n)\}.$$

These are all distinct since $f$ is injective, and therefore this set has $n$ distinct elements.
Since $|B| = n$, this means $f$ is also surjective.

$\boxed{\impliedby}$

Assume there is $f\colon A \to B$ injective.
Since $f$ is injective, this implies $|A| \leq |B|$.
Since $f$ is surjective, this implies $|A| \geq |B|$. (Apply theorem with $A, B$ reversed)
Then $|A| = |B|$.

$\square$