Theorem

Let $f: A \to B$ be a bijection. Then there exists $g: B \to A$ with

$$g(f(x)) = x, \quad \forall x \in A, \qquad f(g(y)) = y \quad \forall y \in B. \tag{2}$$

Proof.

- Assume that $f: A \to B$ is bijective.
- Then for any $y \in B$, there exists a unique $x \in A$ with $f(x) = y$.
- Define $g(y) = x$.
- Then: $g(f(x)) = g(y) = x$.
- And: $f(g(y)) = f(x) = y$.

□