Facts about composition

Theorem

Let $f : A \to B$ and $g : B \to C$.

1. *If f, g are both injective, then $g \circ f$ is also injective.*
2. *If f, g are both surjective, then $g \circ f$ is also surjective.*
3. *If f, g are both bijective, then $g \circ f$ is also bijective.*

Proof.

1. Let $x_1, x_2 \in A$ with $x_1 \neq x_2$.
   Then $f(x_1) \neq f(x_2)$ since f injective.
   Then $g(f(x_1)) \neq g(f(x_2))$ since g injective.
2. Let $z \in C$.
   There exists $y \in B$ with $g(y) = z$ since g surjective.
   There exists $x \in A$ with $f(x) = y$ since f surjective.
   So therefore $g(f(x)) = z$.
3. Put 1 and 2 together!