## Lecture 4, class activity

We state nine theorems below. You are not being asked to prove them now. The idea here is to determine what *strategy* you should employ to try to prove them.

Part I. On your own, try and determine which strategy you would use for each of these. Put the number 1,2,3 in each of the boxes according to the letter of the theorem. Part II. As a class, we will "vote" and discuss what people think.

- 1. Direct proof
- 2. Contraposition, or Proof by contrapositive
- 3. Proof by contradiction

Α	В	С	D	Е	F	G	Н	Ι

- A. Let m, n be odd. Show that mn is odd.
- B. If  $x^2 + 6x 5$  is even, then x is odd.
- C. Let A, B be two square matrices with positive entries. Show that AB has positive entries.
- D. Let  $a, b, n \in \mathbb{Z}$ . Show that if n does not divide the product ab, then n does not divide a and n does not divide b.
- E. Let T be a right triangle with side lengths a, b, c where c is the hypotenuse and a, b are the sides meeting at the right angle. Assume that T is non-degenerate, i.e. that the two legs each have positive length. Prove that a + b > c.
- F. The sum of any three consecutive integers is divisible by 3.
- G. If  $x^2$  is even, then x is even.
- H.  $\sqrt{3}$  is irrational.
- I. There is no smallest positive rational number.