Name:________________________

Lecture 27, class activity. Uncountable sets.

Recall the fact that A^*, the set of all finite strings over the alphabet A, is countable.

1. We say that a decimal expansion is **terminating** if it ends with an infinite sequence of zeros. More specifically, we say that it is terminating if there exists $n \geq 1$ such that for all $k > n$, $d_k = 0$.

   Prove that the set of all terminating decimal expansions is countable.

2. We say that a decimal expansion is **repeating** if $n \geq 1$ such that $d_{k+n} = d_k$ for all $k \geq 1$.

   Prove that the set of all repeating decimal expansions is countable.

   Hint: Is there a way to map (bijectively) repeating decimal expansions to terminating ones?

3. Note that all rational numbers have either a terminating or a repeating decimal expansion[citation needed]. Deduce from this, and the above, that $\mathbb{Q}$ is countable.

4. Now deduce from the fact that $\mathbb{Q}$ is countable and $\mathbb{R}$ is uncountable that the set of irrational numbers, $\mathbb{R} \setminus \mathbb{Q}$, is also uncountable.