Name:__________________________

Lecture 26, class activity. Cardinality.

Let A be a finite set, with $|A| = m$. Let A^n be the set of all vectors of length n with elements in A:

$$A^n = \{(a_1, a_2, \ldots, a_n) : a_i \in A\}.$$

(In CS terms, we might call A an *alphabet* and A^n "the set of strings over A of length n")

A. Compute the size of A^n, and show that it is finite.

B. Now consider the set B^n as the set of strings of length less than or equal to n, or, specifically,

$$\bigcup_{i=0}^{n} A^i.$$

Compute the size of B^n, and again show it is finite.

(This is sometimes called the set of **bounded** strings over A.)

C. Now consider A^*, the set of all finite sequences with elements in A. We will show that A^* is infinite but countable.

- Show that $A^n \subseteq A^*$ and thus $|A^*| \geq |A^n|$. Deduce that $|A^*|$ is infinite.

- Now consider a listing of A^* where we list all sequences of length 0, then length 1, then length 2, etc. Prove that every sequence in A^* is eventually listed in this process, and thus A^* is countable.