Lecture 23, class activity. Well-defined, Part I.

Consider the set \mathbb{R} and $f: \mathbb{R} \to \mathbb{R}$ with $f(x) = x^2$. Define a relation on \mathbb{R} where $x \sim y \iff f(x) = f(y)$. (We showed before that this is an equivalence relation.)

1. Recall that \mathbb{R}/\sim is the set of equivalence classes under \sim ; describe this set, i.e. list all equivalence classes of \sim

2. Now consider the function g(x) = x. Define $G \colon \mathbb{R}/\sim \to \mathbb{R}$ with G([x]) := g(x). Show that G is well-defined.

3. Now consider the function $h(x) = x^3$. Define $H \colon \mathbb{R}/\sim \to \mathbb{R}$ with H([x]) := h(x). Show that H is well-defined.

4. Now consider the function j(x) = x + 1. Define $J : \mathbb{R}/\sim \to \mathbb{R}$ with J([x]) := j(x). Show that J is **not** well-defined.