

Lecture 19, class activity. Relations.

Let us define the relation \mathcal{R} on \mathbb{Z} by

$$a\mathcal{R}b \iff \gcd(a, b) > 1.$$

1. Show that $\mathcal{R}^* = \mathcal{R}$.
2. For which $a \in \mathbb{Z}$ is $(a, a) \in \mathcal{R}$?
3. Let $A_3 = \{n \in \mathbb{Z} : (n, 3) \in \mathcal{R}\}$. Describe the set A_3 .
4. Let $A_6 = \{n \in \mathbb{Z} : (n, 6) \in \mathcal{R}\}$. Describe the set A_6 .
5. Let $A_{25} = \{n \in \mathbb{Z} : (n, 25) \in \mathcal{R}\}$. Describe the set A_{25} .
6. Can you give a compact description of A_m for any integer m ?